

Graduate Student Seminar - Fall 2019

Equivariant cohomology of $\mathbb{Z}/2$ -manifolds

Sergio Chaves, September 27, 2019.

If N is a compact manifold with boundary, then it can be realized as the orbit space of a $\mathbb{Z}/2$ -manifold M ; more precisely, as the quotient M/τ where M is a manifold of the same dimension as N and $\tau: M \rightarrow M$ is an involution. The manifold M might not be unique and it depends on the principal $\mathbb{Z}/2$ -bundles over N . However, the equivariant cohomology module-type of these manifolds is completely determined by the cohomology of N . Before discussing these results, the necessary background on principal bundles and equivariant cohomology will be firstly introduced; only basic notions about singular cohomology will be assumed.

A twisted sheaf proof of Albert's theorem

Félix Baril Boudreau, October 4, 2019

In 1939 A. A. Albert published in *Structures of algebras* a proof that given a central simple algebra A over a field K , A has an involution of the first kind if and only if its class in the Brauer group of K has order 1 or 2.

In this talk we will introduce, following Giraud (1971), a categorical analogue of a principal vector bundle, called *G-gerbe*, as well as the notion of *twisted vector bundle* as defined in the work of Lieblich (2004).

These modern tools give us a completely different point of view on Albert's theorem and allow us to prove this result in an interesting new way.

All the necessary background will be presented and some familiarity with the basic vocabulary of category theory will be assumed.

The Gelfand-Naimark Theorem and Noncommutative Geometry

Luuk Verhoeven, October 11, 2019

In this talk I will talk about the Gelfand-Naimark Theorem, which provides a duality between locally compact Hausdorff spaces and commutative C^* -algebras. We will discuss the proof of this theorem, which has several interesting ingredients, as well as the conceptual implications which form, essentially, the basis of Noncommutative Geometry. If time permits, I will also discuss how my master's thesis fits in this conceptual framework. This talk is intended as a casual introduction, so basic knowledge of functional analysis (norms, duals), topology and complex analysis should suffice. Any extra familiarity with functional analysis (spectra of operators in particular) is beneficial but not necessary.

Sheaves on Posets

Udit Ajit Mavinkurve, October 18, 2019

There is a particularly nice way to interpret posets as topological spaces (via the Alexandroff topology), which allows us to translate many tools from topology to the context of posets. Working over posets, several definitions simplify and become easier to introduce and work with - making posets an important pedagogical tool. In this expository talk, we will see (and hopefully prove) analogues of results such as Quillen's Theorem A. Passing familiarity with basic notions of category theory (e.g. adjunctions) and homological algebra (e.g. injective resolutions, right-derived functors) will be assumed, but not much more beyond that.

Fibrations of Categories

Jarl Taxerås, **Fibrations of Categories**, October 25, 2019

By categorifying the homotopy lifting property of maps between topological spaces, one obtains discrete fibrations between categories. The classical correspondence between (isomorphism classes of) covers of a topological space X and (conjugacy classes of) subgroups of the fundamental group $\pi(X)$, generalizes to an equivalence between the category $DFib(C)$ of discrete fibrations into a category C and presheaves on C . We will see that under this equivalence, universal covers correspond to representable presheaves. Realizing this has some nice consequences, and will give us a curious perspective on some familiar constructions.

What is regularized determinant?

Babak Beheshti Vadeqan, November 1, 2019

Taming infinities has its firm roots throughout the history of mathematics and physics. Euler's work on zeta function and infinite products or Cantor's stunning idea of counting infinities are just two classics among many others. In this talk we will see how one can use zeta function to define the determinant of certain operators -infinite matrices- such as differential operators. Then, if time permits, we will go through Quillen's discovery that connects the regularized determinant to the geometry of some determinant bundles.

Sheaves as Étale Spaces

César Bardomiano Martínez, November 15, 2019

Étale morphisms are certain kind of morphisms between schemes. In general case, a bundle over a topological space X is said to be étale if it is a local homeomorphism. The goal of this talk is to present the equivalence between sheaves on a topological space X and étale spaces. We will give a fairly good amount of details of the ideas involved to prove such equivalence.

Introduction to Toposes and Logic

James Leslie, November 22, 2019

Toposes can be seen as abstractions of the category of sets and functions. We will look at several examples of toposes and discuss some of the logical properties they can exhibit. We will conclude by sketching a topos theoretic proof of the independence of the axiom of choice from a slightly weakened version of Zermelo Frankel set theory.

Schubert Varieties in Partial Flag Manifolds and Generalized Severi-Brauer Varieties

Marios Velivasakis, November 29, 2019

Schubert varieties form one of the most important classes of singular algebraic varieties. They are also a kind of moduli spaces. One problem is that these varieties are not easy to understand and manipulate using only their geometric nature. In this talk, we will discuss about Schubert varieties and present a way to characterize them combinatorially. In addition, we will discuss how they relate to Severi-Brauer varieties $SB(d, A)$ and how we can use their combinatorial description to answer questions about subvarieties of $SB(d, A)$.

Graduate Student Seminar - Winter 2020

Equivariant cohomology of $\mathbb{Z}/2$ -manifolds

Brandon Doherty, January 24, 2020.

Simplicial categories, categories enriched over simplicial sets, are one of the most well-known models for the theory of $(\infty,1)$ -categories. Their homotopy theory is studied by means of a model structure on the category of simplicial categories, due to Bergner. Recently there has been some interest in categories enriched over cubical sets as an alternative model for $(\infty,1)$ -categories. In this talk, I will discuss a model structure on the category of cubical categories which is analogous to the Bergner model structure for simplicial categories. If time permits, I will also discuss an adjunction between the categories of simplicial sets and cubical categories, due to Kapulkin and Voevodsky, and sketch the proof that it is a Quillen adjunction when the category of simplicial sets is equipped with the Joyal model structure.

Hofer's geometry on the group of Hamilton diffeomorphism of the 2-sphere

Prakash Singh, January 31, 2020.

Given an isotopy (or flow of a vector field) on a Symplectic manifold (M, w) , the trajectories of this isotopy may form a complicated system of curves, but

one can study the isotopy as a curve in some topological group ($Ham(M, w)$). In fact, we can put a metric (Hofer's metric) on $Ham(M, w)$ and study the geometric properties of this metric itself. We will look at some basic questions that one can ask about this metric and discuss the diameter of this topological group under this metric. In particular, we will try to prove that $Ham(S^2)$ has infinite diameter with respect to this metric.

Jacobians of graphs and starfish

Mohabat Tarkeshian, February 7, 2020.

Graphs can be viewed as discrete analogues of various mathematical structures, such as algebraic curves and Riemann surfaces. A finite graph has an associated finite abelian group called the Jacobian group, denoted $Jac(G)$ (also called the critical, sandpile, or Picard group). We will discuss these groups and a special type of k -regular graph called a starfish graph. A starfish is similar to a hyperelliptic curve since each can be viewed as a (ramified) double-cover of a simply connected object. It is a graph S exhibited as a double-cover $S \rightarrow T$ of a tree T with ramification locus equal to the leaves of T . We will discuss how varying the starfish can give rise to natural questions such as how often S is Ramanujan and how $S \rightarrow T$ can be viewed as a harmonic morphism of graphs. We will also briefly examine induced maps on Jacobians and their connection to harmonic morphisms.

Monoidal categories, string diagrams, and Markov categories

Luis Scoccola, February 14, 2020.

Monoidal categories are categories with a notion of product that is weaker than the categorical product. This relaxation allows for many interesting examples coming from topology, logic, physics, and probability theory.

I will introduce monoidal categories and string diagrams, a graphical calculus to work with diagrams in monoidal categories, and I will go over several examples, paying particular attention to the ones coming from probability theory. This will motivate the definition of Markov category, an abstraction that lets us study probabilistic concepts such as marginalization, conditionals, Markov processes, almost sure equality, and deterministic maps diagrammatically.

Arithmetic Dynamics: Uniform Boundedness, and two Galois Representations

Andrew Herring, February 28, 2020.

Arithmetic dynamics is a relatively new area of research in which we ask number theoretic questions about dynamical systems. An easy source of problems is by

analogy with arithmetic geometry: starting from some problem in arithmetic geometry (solved or not), we can consider its dynamical analogue (for example by replacing the words "torsion points on an abelian variety" with the words "(pre)periodic points of a rational map.") In this talk we'll look at the (dynamical) uniform boundedness conjecture of Morton and Silverman (an analogy of Merel's Theorem on the uniform bound on torsion of an elliptic curve), and two Galois representations-the arboreal Galois representation, and the dynatomic Galois representation, each of which might be regarded as an analogy of the l-adic representation of an abelian variety. This talk is meant to be a gentle introduction to some of the basic problems in arithmetic dynamics; the only pre-requisites are an open heart and an open mind.

TBA

Jeremy Gamble, March 6, 2020.

TBA

Alejandro Santacruz, March 13, 2020.

TBA

Nathan Joseph Pagliaroli, March 20, 2020.

TBA

Luke Broemeling, March 27, 2020.